Maximum Directivity Beamformer for Spherical-Aperture Microphones

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Abstract
Performance of microphone arrays at the high-frequency range is typically limited by aliasing, which is a result of the spatial sampling process. A potential approach to avoid spatial aliasing is by using continuous sensors, in which spatial sampling is not required. The proposed beamforming technique is used to compute the optimal real-valued aperture weighting function for a spherical-aperture microphone. Real-valued aperture weighting functions are required to realize the directivity of the sensor.

Spherical microphone arrays

Drawbacks:
• Spatial aliasing, limited high-frequency performance
• Highly directional microphone arrays require many microphones and an expensive system for connecting the microphones

Advantages:
• Enables 3D processing
• Beamforming, create high-directivity beam patterns
• Does not suffer from spatial aliasing
• Could replace systems with a large number of sensors, reduced realization cost

Drawbacks:
• Weighting cannot be applied with simple electronics
• Real-valued weighting is required to ensure realizability

A practical method for embedding weight function - varying electrode density [Francois et al., 2003]:

Spherical aperture microphone

Formulation:

\[ f_{in} = \int_{0}^{2\pi} \int_{0}^{\pi} f(\theta, \phi) Y_n^m(\theta, \phi) \cos \phi \, d\phi \, d\theta \]

\[ f(\theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} f_{nm} Y_n^m(\theta, \phi) \]

The spherical-aperture microphone output [Rafaely, 2005a]:

\[ y(k, r) = \int_{0}^{2\pi} \int_{0}^{\pi} p(k, r, \theta, \phi) w^* \theta, \phi) \cos \phi \, d\phi \, d\theta \]

\[ \rho_{nm} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \rho_{nm}(k, r) w_{nm} \]

\[ \rho_{nm} \text{ and } w_{nm} \text{ - Spherical Fourier transform of } p \text{ and } w \text{ respectively.} \]

Assumptions [Rafaely, 2005a]:
1. Single plane wave
2. Unit amplitude
3. Known direction of arrival
4. Symmetrical beam pattern around the look direction
5. Mode gains for a rigid sphere
6. Plane wave direction of arrival
7. Look direction - microphone look direction.
8. Acts as the only parameter in the weighting function, must be real

Aperture weighting function, and microphone output:

\[ w(\theta) = \sum_{n=0}^{\infty} d_n Y_n^0(\theta) \]

\[ y(k, r, \theta, \phi) = \sum_{n=0}^{\infty} d_n b_n(\theta) P_n(\cos \theta) \]

References

Maximum Directivity Beamformer

Finds the optimal coefficients \( d_n \) that maximize the directivity of a spherical-aperture microphone with a given order of \( N \). Directivity [Rafaely, 2005b]:

\[ Q = \frac{d^T b^T b d}{d^T C d} \]

\[ C = (d^T)^2 \text{disq} \left( b_0^2, 3 b_1^2, ..., (2N+1) b_N^2 \right) \]

\[ d - \text{weight coefficients vector} \]

\[ b - \text{look-direction steering vector} \]

Formulation:

\[ \mathbf{d}_{\text{opt}} = \arg \max_{\mathbf{d}} Q \text{ s.t. } |\mathbf{d}| = 1 \]

Simplifying the problem formulation:

\[ \mathbf{d}_{\text{opt}} = \arg \min_{\mathbf{d}} (d^T C d) \text{ s.t. } d^T b^* d = 1 \]

Solution for this problem is known... but complex:

\[ \mathbf{d}_{\text{opt}} = \frac{\mathbf{C}^{-1} \mathbf{b}}{\mathbf{b}^T \mathbf{C}^{-1} \mathbf{b}} \]

A real-valued solution does not exist, and needs to be fully derived.

Using Lagrange multipliers method:

\[ J = d^T C d + \lambda \left( d^T b^* d - 1 \right) \]

Taking the gradient of \( J \) with respect to \( d \) and setting to zero:

\[ d^T C + \lambda d^T \text{Re} \{ b b^* \} = 0 \]

Imposing the constraint and simplifying:

\[ d_{\text{opt}} = \frac{C_c}{c^T C_c^{-1}} \cdot c = \text{Re} \{ b e^{-\frac{1}{b^* b}} \} \]

Simulation Examples

Figure: (a) narrowband performance and sensitivity (b) broadband performance

Figure: Maximum directivity beamformer with bounded sensitivity

Conclusions
• Initial feasibility of spherical-aperture microphones was shown.
• No spatial aliasing.
• Easier realization than equivalent spherical microphone array.
• Simple beamforming techniques.
• Aperture weighting by varying electrode density.

Plane wave response

Spherical Fourier Transform [Driscoll & Healy, 1994]:

\[ f_{in} = \int_{0}^{2\pi} \int_{0}^{\pi} f(\theta, \phi) Y_n^m(\theta, \phi) \sin \theta \, d\phi \, d\theta \]

\[ f(\theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} f_{nm} Y_n^m(\theta, \phi) \]

The spherical-aperture microphone output [Rafaely, 2005a]:

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\[ P_n \text{ - Legendre polynomial of order } n \]

\[ \theta - \text{ spatial angle between the } \Omega_0 \text{ and } \Omega_1 \]